

# A Model for Predicting the Thermal Conductivities of Bentonite-Bonded Molding Sands at High Temperatures

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The effective thermal conductivities of bonded molding sands vary with the dry density, binder content, initial moisture content, temperature as well as the types of sand and binder clay. In this study, a theoretical model for predicting the effective thermal conductivities of bentonite-bonded molding sands was developed. The results of measurement of the effective thermal conductivities of molding sands at temperatures up to 750°C were used. The binder thermal conductivities of both western bentonite and southern bentonite were suggested as a function of dry density, binder content and initial moisture content and were assumed not to vary with temperature. The radiation model proposed by Vortmeyer was also incorporated. The model developed in this study was proved to predict well the effects of binder content, initial moisture content, dry density and temperature.

**Key Words:** Thermal Conductivity, Molding Sand, Bentonite Clay Binder, Theoretical Model, High Temperature.

## Nomenclature

$a+2s$  : Constants in Eq. (4) [ $m^{-1}$ ]  
 $B$  : Binder content by weight  
 $k_{be}$  : Effective thermal conductivity of binder [W/mK]  
 $k_c$  : Thermal conductivity due to conduction [W/mK]  
 $k_e$  : Effective thermal conductivity of molding sand [W/mK]  
 $k_f$  : Thermal conductivity of saturating fluid [W/mK]  
 $k_{fr}$  : Thermal conductivity due to intergranular radiation [W/mK]  
 $k_s$  : Thermal conductivity of sand grain [W/mK]  
 $k_{se}$  : Effective thermal conductivity of sand grain [W/mK]  
 $k_{sr}$  : Thermal conductivity due to transgranular radiation [W/mK]

$M$  : Initial moisture content by weight  
 $r$  : Variable in the model of Fig. 1  
 $r_b$  : Variable in the model of Fig. 1  
 $R$  : Characteristic radius in the model of Fig. 1  
 $x$  : Coordinate  
 $y$  : Coordinate  
 $\rho_d$  : Dry density [ $g/cm^3$ ]  
 $\phi_b$  : Volume fraction of binder  
 $\phi_f$  : Volume fraction of fluid in the pore space  
 $\phi_s$  : Volume fraction of sand particles

## 1. Introduction

The earliest experimental investigation of the thermal properties of molding sands was carried out by Briggs and Gezelius (1933). Atterton (1953) investigated the influences of several parameters on the effective thermal conductivity of bonded sand molds.

Whitmore and Ingerson (1960) proposed an expression to predict the effective thermal conductivity of clay-bonded silica molding sands. Kubo et al. (1982) developed a heat transfer model

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modified from the Kunii's theory for a packed bed. A model equation was suggested for the prediction of the effective thermal conductivity of sand molds at temperatures below 600°C. The predictions show good agreement with the measured thermal conductivities of dry molding sands bonded with western bentonite and of resin-bonded molding sands. Kubo et al. (1983) measured the thermal properties of dry and green molds of silica, olivine, zircon and chromite sands by the pouring method. The effective thermal conductivity as a function of temperature was obtained from the pouring method using a parameter optimization technique.

Hartley and Patterson (1983) investigated the effects of temperature, initial moisture content, and binder content on the effective thermal conductivity of bentonite-bonded silica and zircon sand molds. From the measurements at temperatures up to 750°C, they presented the normalized functions to account for the effects of temperature, initial moisture contents and binder content.

Park and Hartley (1992) developed a theoretical model for predicting the effective thermal conductivities of unbonded sands and sands bonded with liquid binders. Also, Park and Hartley (1996) measured the thermal conductivities of bonded molding sands at temperatures up to 750°C using the line-heat-source method.

In this study, a theoretical model for predicting the effective thermal conductivities of bonded molding sands at temperatures up to 750°C was developed by modifying the model developed by Park and Hartley (1992). Also, the predicted results from the model were compared with the measured results reported by Park and Hartley (1996). The model predictions were proved to be reasonably accurate and thus it could be used to examine the effects of the dry density, binder content, initial moisture content and temperature when either western bentonite or southern bentonite was used as binder.

## 2. Model Development for Bonded Molding Sands

A model was developed by Park and Hartley

(1992) to predict the effective thermal conductivities of unbonded sands and sands bonded with liquid binders. This model was not applied to clay-bonded molding sands because of the different characteristics of clay binder. The difference between bentonite clay binders and liquid binders can be attributed to the complex bonding mechanism of bentonite clay in molding sands. Bentonite clay forms a film on the surface of sand particles and a porous bond at the contact points between the sand particles. The effectiveness of bentonite clay is much reduced to enhance the heat transfer through the bentonite bond between the sand particles.

The bonding structure of bentonite clay in molding sands has been investigated using a scanning electronic microscope. But no detailed analysis is available to distinguish the influence of the bentonite binder on the effective thermal conductivities of bentonite-bonded molding sands.

The effect of bentonite clay binder on the effective thermal conductivities of molding sands is included in the model for liquid binders by assuming that all of the bentonite clay coalesces at the contact points between the sand particles just as liquid binders do in the unsaturated sands. With this assumption, the effective thermal conductivity of dried bentonite bond in the molding sands could be calculated from the model using the measured effective thermal conductivity of bonded molding sands reported by Park and Hartley (1996). Thus, the model developed by Park and Hartley (1992) can be used for the bentonite-bonded molding sands also and is shown in Fig. 1.

The model is composed of three phases. The shape of sand assumed is a cylinder with a side end having spherical surface. The binder was assumed to coalesce at the contact point between the sand particles. The effective thermal conductivity of western bentonite binder determined as described above could be expressed as a function of apparent density of the bonded sand and the ratio of initial moisture content to the binder content by weight. Least-square curve fits of the calculated effective thermal conductivities of

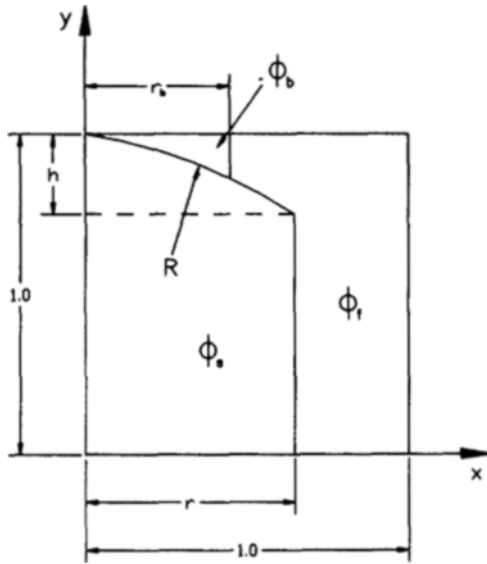


Fig. 1 Model geometry

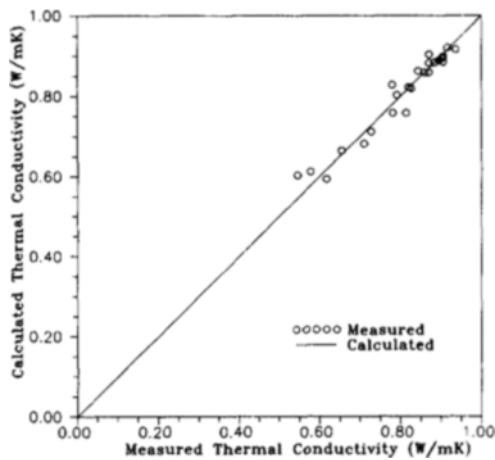


Fig. 2 Comparison between the predicted and the measured thermal conductivities of Ottawa sands bonded with western bentonite.

western bentonite binder in bonded Ottawa sands results in Eq. (1).

$$k_{be} = 0.03322 \ln(M/B) + 0.1494\rho_d - 0.1016 \quad (1)$$

With this functional relationship, the effective thermal conductivity of Ottawa sand bonded with western bentonite can be determined from the model.

The validity of Eq. (1) was evaluated by

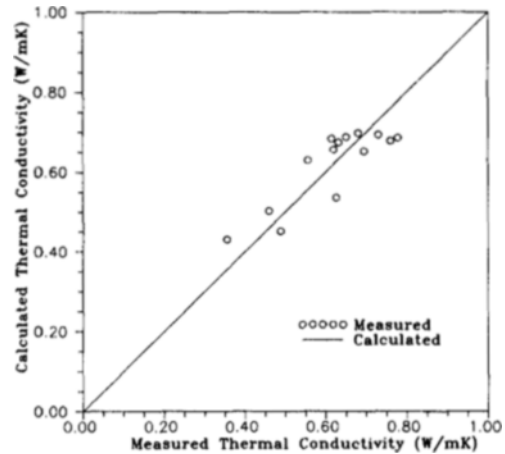


Fig. 3 Comparison between predicted and measured thermal conductivities of Ottawa sands bonded with southern bentonite.

predicting the effective thermal conductivity of bentonite-bonded Ottawa sand from the model using Eq. (1). The calculated values are compared with the measured values reported by Park and Hartley (1996) in Fig. 2.

In the measurements, both the initial moisture content and the binder content were varied from 0.02 to 0.08 and the dry density was varied from 1.5 to 1.7 g/cm<sup>3</sup> for both bentonite types. The comparison shows relatively good agreement with an error less than 10%.

The thermal conductivity of southern bentonite clay binder in molding sands was calculated from the measured effective thermal conductivity of Ottawa sand bonded with southern bentonite. The resulting effective thermal conductivity of southern bentonite binder was also determined as a function of the ratio of initial moisture content to binder content by weight and the dry density as Eq. (2).

$$k_{be} = 0.01032 \ln(M/B) + 0.1364\rho_d - 0.1342 \quad (2)$$

The effective thermal conductivity of Ottawa sand bonded with southern bentonite is calculated using the model with the thermal conductivity obtained from Eq. (2). The calculated values are compared with the measured values in Fig. 3, which shows relatively good agreement with an

error less than 25%.

### 3. Thermal Conductivity at High Temperatures

At high temperatures, the prediction of the effective thermal conductivity of molding sand is complicated because the radiation heat transfer may become significant above 500°C and the thermal conductivities of the components in molding sands may change with temperature. Two different radiation heat transfer mechanisms may exist in molding sands: the intergranular radiation across the pore spaces and the transgranular radiation through the sand grains.

Several models have been proposed to describe the radiation heat transfer coupled with the conduction heat transfer in heterogeneous materials. Vortmeyer (1978) considered the specimen as a pseudo-homogeneous material, and the radiative and conductive heat transfer mechanisms are assumed to act in parallel with one another. Thus, the effective thermal conductivity of the material is written as Eq. (3).

$$k_e = k_c + k_{fr} \quad (3)$$

where

$$k_{fr} = 4 n \sigma T^3 D$$

Here,  $\sigma$  is a Stefan-Boltzmann constant,  $n$  is a constant whose value depends upon the emissivity and the geometry of the surface of the pore space,  $T$  is the absolute temperature, and  $D$  is the size of the pore space.

The radiation through the sand particles depends on the optical properties of the sand particles. The equation for the relation between the temperature and the radiant heat flux densities is nonlinear. An approximate solution was developed by Chen (1963) for a plate having two constant-temperature boundaries. When the material is optically thick, the radiant thermal conductivity for locations far from the boundaries can be approximated by Eq. (4).

$$k_{sr} = 8\sigma T^3 / (a + 2s) \quad (4)$$

This expression was presented by Hamaker (1947) and also by Chen and Churchill (1963).

Whitmore and Ingerson (1960) suggested an empirical expression by assuming that the term  $(a + 2s)$  varied exponentially with the temperature as Eq. (5).

$$a + 2s = \alpha \exp(-\beta T) \quad (5)$$

where  $T$  is the absolute temperature in  $K$ , and  $\alpha$  and  $\beta$  are 148800 and 0.003852, respectively, having been determined from the experimental values of the radiant thermal conductivity for silica sand.

Radiation can be treated as an additive term to the conduction heat transfer as considered by Whitmore and Ingerson (1960). Then the effective thermal conductivity of the sand grain can be determined from Eq. (6).

$$k_{se} = k_s + k_{sr} \quad (6)$$

Thus, the model for predicting the effective thermal conductivity of molding sands at high temperatures was developed by including the radiation heat transfer mechanism to the model of Park and Hartley (1992). The expression for the model shown in Fig. 1 to predict the thermal conductivity of bonded molding sands can be written as Eq. (7).

$$k_e = \frac{2}{\frac{1}{k_{se}} - \frac{1}{k_{be}}} \left[ a_b \ln \left( \frac{\sqrt{R^2 - r_b^2} + a_b}{R + a_b} \right) - \sqrt{R^2 - r_b^2} + R \right] + \frac{2}{\frac{1}{k_{se}} - \frac{1}{k_f}} \left[ a_f \ln \left( \frac{\sqrt{R^2 - r^2} + a_f}{\sqrt{R^2 - r_b^2} + a_f} \right) - \sqrt{R^2 - r^2} + \sqrt{R^2 - r_b^2} \right] + k_{fr} \quad (7)$$

where

$$a_b = \frac{(1-R)/k_{se} + R/k_{be}}{1/k_{se} - 1/k_{be}}$$

$$a_f = \frac{(1-R)/k_{se} + R/k_f}{1/k_{se} - 1/k_f}$$

The contributions of the heat transfer mechanisms existing in bonded Ottawa sand at high temperatures were calculated by using the model and shown in Fig. 4.

Curve A in Fig. 4 represents the variation of the effective thermal conductivity of bonded Ottawa sand with temperature when only the variation of thermal conductivity of the silica

sand particles with temperature is considered. The thermal conductivity of air is assumed equal to its room temperature value in this calculation. The effective thermal conductivity decreases with increasing the temperature because the thermal conductivity of the silica sand particles decreases with the temperature. This figure shows that the variation of the thermal conductivity of the sand particles is the most important parameter in the temperature range considered.

The influence of the temperature dependence of the thermal conductivity of air is included in curve B. Thus, curve B represents the effective thermal conductivity of molding sand owing to the conduction only. This curve shows that the temperature dependence of the thermal conductivity of air is significant and results in an increase in the effective thermal conductivity of about 14 percent at 750°C.

Curve C represents the calculated effective thermal conductivity including the effect of the radiation heat transfer across the air gap and pores. The contribution of intergranular radiation is about 8 percent at 750°C. The total effective thermal conductivity is shown in curve D which also includes the effect of the radiation heat transfer through the sand grains. This curve shows that the effect of transgranular radiation in molding sands is very small below 750°C, and its contribution to the total effective thermal conductivity is about 2 percent.

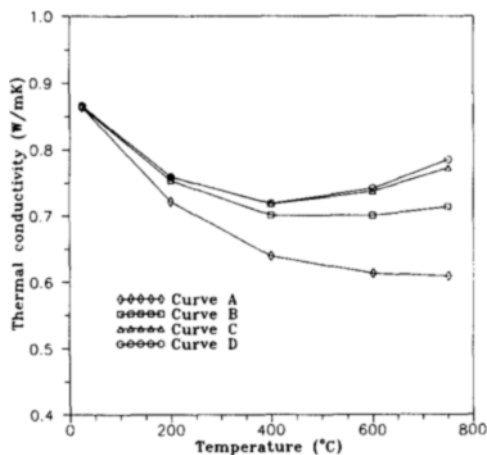


Fig. 4 Effect of various heat transfer mechanisms.

Figure 4 shows that the radiation heat transfer becomes significant above 500°C and the contribution to the effective thermal conductivity is only 10 percent at 750°C. However, the effect of the radiation can increase rapidly at higher temperatures.

In Fig. 5, the measured effective thermal conductivities of Ottawa sands bonded with western and southern bentonite clays are compared with the calculated results from the model. The values predicted by the model agree reasonably well with the measured values for both binder types.

#### 4. Theoretical Model Prediction

The bentonite clay bond cannot be treated as a liquid binder because, as pointed out by Hartley (1981), the bentonite bond in dry molding sands has a porous structure, and the sand particles are coated with a film of bentonite clay. Therefore, the enhancement of the conduction through molding sand provided by the bentonite bond is significantly less than that provided by liquid binders.

The effective thermal conductivity of Ottawa sand bonded with western bentonite was calculated as a function of the binder content by weight using the model. The calculated results are shown in Fig. 6.

This figure shows that, as the binder content increases, the effective thermal conductivity of

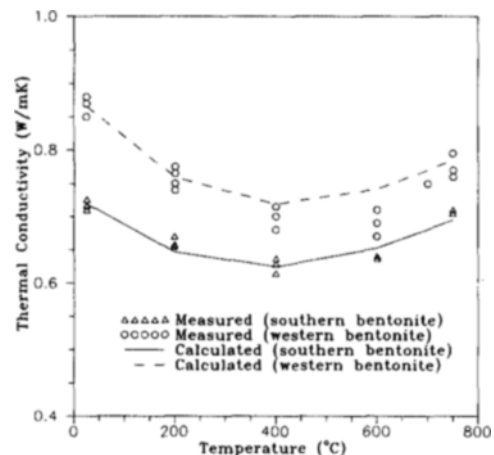


Fig. 5 Comparison of the thermal conductivities of bonded Ottawa sand at high temperatures.

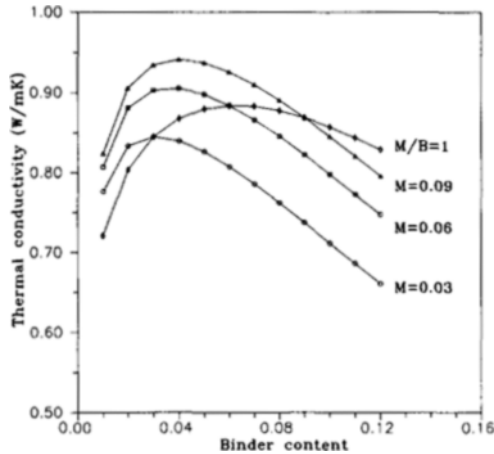


Fig. 6 Effect of binder content for Ottawa sand bonded with western bentonite

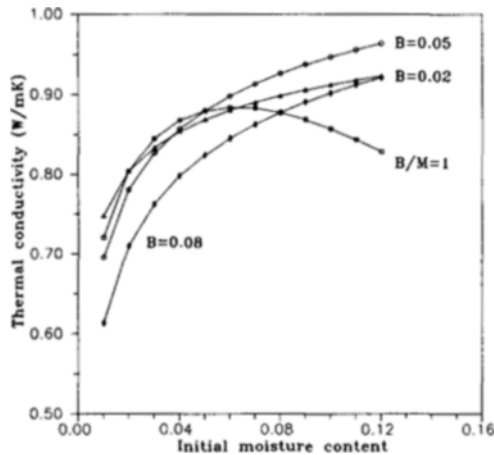


Fig. 7 Effect of initial moisture content for Ottawa sand bonded with western bentonite

molding sand increases until an optimum binder content is reached and decreases thereafter. As the binder content increases, the heat transfer area through the binder film increases rapidly at low binder contents. However, the ratio of initial moisture content to binder content decreases, the resulting effective thermal conductivity of the binder will decrease. Therefore, an optimum binder content exists when the initial moisture content and the dry density are maintained constant.

This figure also shows the variation of the effective thermal conductivity with binder content when the ratio of initial moisture content to

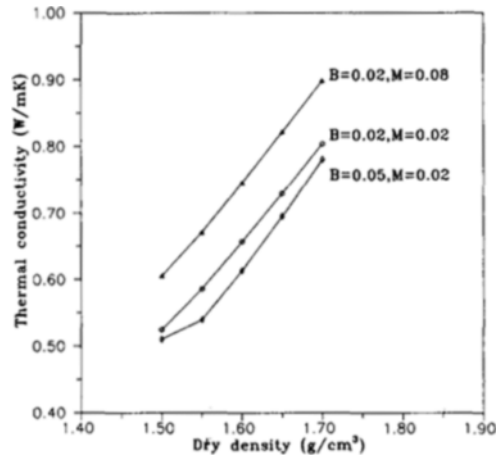


Fig. 8 Effect of dry density for Ottawa sand bonded with western bentonite

binder content is unity. Thus the effective thermal conductivity of the bentonite bond does not change with the binder content. The difference between the conductivity values for a constant ratio of initial moisture content to binder content and the conductivity value for the same sand with a constant initial moisture content represents the effect of the variation of the effective thermal conductivity of the bentonite bond to different initial moisture content.

The effect of the initial moisture content on the effective thermal conductivity of bonded molding sand is shown in Fig. 7.

When the dry density is maintained constant, the effective thermal conductivity increases with increasing the initial moisture content. This indicates that the addition of moisture to the molding sand causes an increase in the effective thermal conductivity of bentonite bond and thus its effect is more significant at greater binder contents.

Figure 8 shows the calculated effective thermal conductivity of Ottawa sand bonded with western bentonite as a function of dry density.

For a constant initial moisture content and constant binder content, the effective thermal conductivity increases almost linearly with increasing the the dry density. This is probably a result of a decrease in the thickness of the binder film between the sand particles with an increase in the dry density. This figure also shows that the

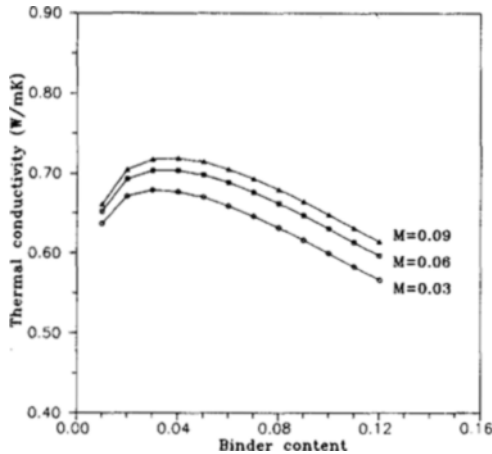


Fig. 9 Effect of binder content for Ottawa sand bonded with southern bentonite

dry density affects the effective thermal conductivity much more than does the initial moisture content or the binder content for the range of values used in this study.

The effect of the binder content on the effective thermal conductivity of bonded molding sands are quite different when different bentonite types are used.

Figure 9 indicates that the effective thermal conductivity at a constant initial moisture content and a constant dry density increases until an optimum binder content is reached and decreases thereafter. The effect of the binder content on the effective thermal conductivity is less significant with southern bentonite than with western bentonite, and the effective thermal conductivity of molding sands is lower when southern bentonite is used.

Figure 10 shows the variation of effective thermal conductivities of Ottawa sand bonded with western and southern bentonite with temperature at various dry density.

In this figure, the initial moisture content and the binder content is maintained at 5 and 6 percent, respectively. As the dry density increases, the temperature at which the minimum value of the effective thermal conductivity occurs increases, and the effect of temperature is more significant for both bentonite types. At higher dry densities, the clay film between the sand particles

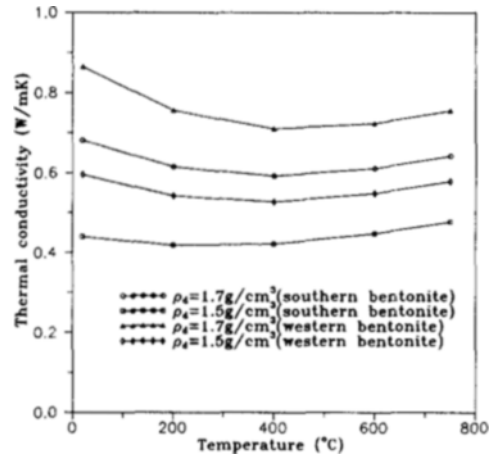


Fig. 10 Effect of temperature for bentonite-bonded Ottawa sands.

is thinner and the conductance of the clay film between the sand particles is higher. Thus, the effect of the decrease in the thermal conductivity of sand particles with temperature is more significant. However, at low dry densities, the effects of an increase in the thermal conductivity of air and an increased contribution of the radiation heat transfer are more significant because the contributions of these heat transfer mechanisms are greater in bonded sands having lower dry densities.

## 5. Conclusions

The following conclusions are drawn from the results of this study.

(1) The model developed in this study can predict reasonably well the effect of binder content, initial moisture content, dry density, and temperature for molding sands bonded with western and southern bentonite types.

(2) At temperatures up to 750°C, the contributions of the conduction through sand particles and pore spaces, the transgranular radiation and the intergranular radiation are analyzed.

(3) The optimum binder content exists when the dry density and the initial moisture content are maintained constant for western and southern bentonite clays.

(4) The effect of the dry density is more significant than the binder content and the initial

moisture content in the range used in this study.

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